

Meta-analytic structural equation modelling

Application to Mendelian randomization studies

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Outline



- Meta-analytic structural equation modelling
- SEMs RAM specification
- linear IV single study and meta-analysis (two-stage structural equation modelling)
- · Mediation model with IVs single study and meta-analysis
- Conclusion

Meta-analytic structural equation modelling



- Studies report cov/corr matrices between variables (N, \overline{x})
- SEMs and causal inference: Vanderweele (2012), De Stavola *et al.* (2015)
- SEMs for MR: Shin *et al.* (2014); Burgess *et al.* (2015); Evans & Davey Smith (2015); Ziegler *et al.* (2015)
- Cheung & Chan (2005); Meta-analytic structural equation modeling: A two stage approach
- Cheung (2015a) Meta-analysis: A structural equation modeling approach.
- metaSEM package for R (Cheung, 2015b); calls OpenMx (Boker et al., 2011)

SEMs and meta-analysis ^{Mathematics} Lancaster & Statistics University

- Using GLMM commands to fit meta-analysis models:
 - 1. random effects numerical integration
 - 2. constraints especially on residual variance correct SEs
 - Normand (1999) SAS proc mixed
 - Van Houwelingen et al. (2002) SAS proc mixed
 - Stijnen et al. (2010) SAS proc nlmixed
 - Bagos (2015) Stata gllamm
- Using SEM commands to fit meta-analysis models:
 - Cheung (2008) Mplus
 - Cheung (2010) LISREL
 - Cheung (2013) R metaSEM, OpenMx; MPlus
 - Palmer & Sterne (2015) Stata sem, gsem

SEMs: RAM specification ^{Mathematics} Lancaster & Statistics

- SEM specifications: LISREL, EQS, Mplus, RAM (OpenMx & metaSEM)
- 4 matrices *F*, *M*, *A*, and *S*, and a vector *v*
- v represents all variables in the model.
- **A** specifies regression coefficients and factor loadings (single headed arrows).
- *S* var-covar matrix or corr matrix of variables (double headed arrows).
- *M* means of the variables.
- *F* selection matrix indicating the observed variables.

RAM speicification II



• SEM implied means, $\mu(\theta)$, and covariance matrix $\Sigma(\theta)$, where I is an identity matrix, are

$$\mu(\theta) = \boldsymbol{F}(\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{M}$$

$$\Sigma(\theta) = \boldsymbol{F}(\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{S}((\boldsymbol{I} - \boldsymbol{A})^{-1})'\boldsymbol{F}'.$$

Mendelian randomization & Statistics University University

(Full information) ML estimation of linear instrumental variable
 model

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(Full information) ML estimation of linear instrumental variable
 model



Mendelian randomization A Statistics Lancaster with University University

(Full information) ML estimation of linear instrumental variable
 model



Path diagram (Burgess *et al.*, 2015, Figure 4)



Mendelian randomization & Statistics University University

(Full information) ML estimation of linear instrumental variable
 model



Example fit in OpenMx I Mathematics Lancaster

Simulate a single study

n = 10000G = Binomial(2, p = 0.3) $Y = 1 + X + \varepsilon_1$ $X = 1 + G + \varepsilon_2$. $\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \sim MVN \left(0, \ \Sigma = \begin{bmatrix} 1 & 0.75 \\ 0.75 & 1 \end{bmatrix} \right)$

• Observed means and covariances (first stage $R^2 = 0.3$)

Means = $\begin{bmatrix} y : 2.60 \\ x : 1.61 \\ g : 0.61 \end{bmatrix}$ Covariances = $\begin{bmatrix} 3.92 & 2.17 & 0.43 \\ 2.17 & 1.43 & 0.42 \\ 0.43 & 0.42 & 0.42 \end{bmatrix}$





$$\boldsymbol{A} = \begin{bmatrix} 0 & \beta_1 & 0 \\ 0 & 0 & \alpha_1 \\ 0 & 0 & 0 \end{bmatrix}, \ \boldsymbol{S} = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & \sigma_{\varepsilon_1 \varepsilon_2} & 0 \\ \sigma_{\varepsilon_1 \varepsilon_2} & \sigma_{\varepsilon_2}^2 & 0 \\ 0 & 0 & \sigma_g^2 \end{bmatrix}, \ \boldsymbol{M} = \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \bar{g} \end{bmatrix}, \ \boldsymbol{F} = \boldsymbol{I}(3)$$

library(OpenMx)

```
# define model
tsls.model <- mxModel("TSLS", mxData(observed=my.cov, type="cov",
    numObs=n, means=my.means), A1, S1, M1, F1,
    mxExpectationRAM(A="A1", S="S1", F="F1", M="M1",
    dimnames = c("y","x","g")), mxFitFunctionML())</pre>
```

```
# fit model
tsls.fit <- mxRun(tsls.model)
if (tsls.fit$output$status$code <= 1) summary(tsls.fit)</pre>
```

Example fit in OpenMx III ^{Mathematics} Lancaster ^{Sector} Lancaster

> summary(tsls.fit)
Summary of TSLS

free parameters:

	name	matrix	row	СС	1	Estimate	Std.	.Error	r A		
1	beta1	A1	1	2	1	.0018335	0.01	529119	92		
2	alpha1	A1	2	3	1	.0064283	0.01	539188	35		
3	errory	S1	1	1	0	.9987171	0.026	682217	73		
4	errorxy	S1	1	2	0	.7455682	0.019	971792	28		
5	errorx	S1	2	2	0	.9991108	0.014	413014	15		
6	varg	S1	3	3	0	.4218099	0.00	596560	00		
7	beta0	M1	1	У	0	.9870826	0.026	657724	12		
8	alpha0	M1	1	x	0	.9950869	0.013	372953	30		
9	meang	M1	1	g	0	.6115000	0.000	649469	91		

• Estimation using standardised variables (correlations) $\beta_{1s} = \beta_1 \frac{sd(x)}{sd(y)} = 1.002 \times 1.19/1.98 = 0.604$



Approaches:

- Fit TSLS in each study and then meta-analyse
- Estimate numerator (YG) and denominator (XG) in each study, meta-analyse, then calculate ratio
- Fit SEM to all studies (parameters constrained equal across studies, e.g. Stata ssd framework)
- METASEM:
 - 1. Meta-analyse corr/covs (GLS) then weighted SEM
 - 2. TSSEM: multi-group SEM then weighted SEM





- Stage 1: estimate pooled correlations/covariances using a multigroup SEM
- Correlations/covariances constrained equal in each study
- Estimation weighted by the var-covar matrices of each study's corr/cov matrix







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• Fit SEM using ADF (GMM) estimation weighted by inverse of var-cov matrix of pooled correlation matrix

 $r_{F} = \text{vechs}(\text{Stage 1 pooled cov/corr matrix})$ $V_{F} = \text{Var-covar matrix of Stage 1 pooled cov/corr matrix}$ $\rho(\theta) = \text{vechs}(\Sigma) = \text{vechs}(F(I - A_{\theta})^{-1}S_{\theta}((I - A_{\theta})^{-1})'F')$ $\text{ADF minimises:} (r_{F} - \rho(\theta))'V_{F}^{-1}(r_{F} - \rho(\theta))$

TSSEM example



- Simulate 5 studies from same model ($N = 1000, \overline{R}^2 = 0.3$)
- Fit TSLS in each study and meta-analyse

Unstandardised variables



TSSEM example



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- Fit TSLS in each study and meta-analyse

Unstandardised variables Standardised variables



TSSEM example Stage 1: covariances



```
> fixed1 <- tssem1(scovs, sampsizes, method="FEM", cor.analysis=FALSE)</pre>
> summary(fixed1)
. . .
Coefficients:
       Estimate Std.Error z value Pr(>|z|)
S[1,1] 4.0290830 0.0806222 49.975 < 2.2e-16 ***
S[1,2] 2.2430250 0.0467777 47.951 < 2.2e-16 ***
S[1.3] 0.4400050 0.0191257 23.006 < 2.2e-16 ***
S[2.2] 1.4639978 0.0292948 49.975 < 2.2e-16 ***
S[2,3] 0.4246214 0.0124470 34.114 < 2.2e-16 ***
S[3,3] 0.4054322 0.0081127 49.975 < 2.2e-16 ***
___
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
. . .
> coef(fixed1)
        y
            x
v 4.029083 2.2430250 0.4400050
x 2.243025 1.4639978 0.4246214
g 0.440005 0.4246214 0.4054322
```

TSSEM example Stage 2: covariances



```
> fixed2 <- tssem2(fixed1. Amatrix=A1. Smatrix=S1. Fmatrix=F1.</p>
+ diag.constraints=TRUE, intervals.type="LB")
> summarv(fixed2)
95% confidence intervals: Likelihood-based statistic
Coefficients:
     Estimate Std.Error lbound ubound z value Pr(>|z|)
beta1 1.03623
                   NA 1.01956 1.05249
                                        NΑ
                                                ΝA
alpha1 1.04733
                   NA 1.02950 1.06517
                                        NΑ
                                                NΑ
      0.95251
                   NA 0.90865 0.99738 NA
                                                NΔ
ev
exv
      0.72599
                   NA 0.68835 0.76407
                                        NΑ
                                                ΝA
      1.01928
                   NA 0.97924 1.05927
                                        NA
                                                NA
ex
```

• FE MA of TSLS estimates: 1.04 (1.00, 1.08)

TSSEM example Stage 1: correlations



```
> fixed1s <- tssem1(sscovs, sampsizes, method="FEM", cor.analysis=TRUE)
> summary(fixed1s)
Coefficients:
       Estimate Std.Error z value Pr(>|z|)
S[1,2] 0.9235678 0.0020814 443.722 < 2.2e-16 ***
S[1,3] 0.3446074 0.0124713 27.632 < 2.2e-16 ***
S[2,3] 0.5514068 0.0098498 55.982 < 2.2e-16 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
. . .
> coef(fixed1s)
         sv
                 SX
                          sg
sv 1.0000000 0.9235678 0.3446074
sx 0.9235678 1.0000000 0.5514068
```

sg 0.3446074 0.5514068 1.0000000

TSSEM example Stage 2: correlations



 Estimate Std.Error Ibound ubound z Value Pr(>|z|)

 betal 0.6249602 0.0122872 0.6008779 0.6490426 50.863 < 2.2e-16 ***</td>

 alphal 0.5514068 0.0116996 0.5284761 0.5743376 47.130 < 2.2e-16 ***</td>

 ey 0.2361890 0.0086635 0.2192090 0.2531691 27.263 < 2.2e-16 ***</td>

 exy 0.2986075 0.0114840 0.2760993 0.3211157 26.002 < 2.2e-16 ***</td>

 ex 0.6959505 0.0129382 0.6705921 0.7213089 53.790 < 2.2e-16 ***</td>

 --

 Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

• • •

- FE MA of standardised TSLS: 0.63 (0.60, 0.65)
- Convert to unstandardised scale: β₁ = β_{1s} sd(y)/sd(x) need average sd(x) and sd(y) to rescale

Mediation model: concept^{Mathematics} Lancaster



- Concerns about possible measurement error/confounding in the mediator
- Cole & Preacher (2014) outline problems using imprecisely measured variables in path analysis:
 - Under or over-estimate effects
 - Reduced power
 - Valid models appear invalid
 - Differential measurement error in various parts of model can change conclusions
 - Problems get worse as models get more complex
- Instrument the mediator as well as the exposure
- Similar model proposed in clinical trials (Imai et al., 2013)
- Non-parametric SEM / 3SLS estimation (Tchetgen Tchetgen & Lin, 2012)

Mediation model: definition



DAG Relton & Davey Smith (2012)





B Step 2



Mediation model: definition



DAG Relton & Davey Smith (2012)



B Step 2





Mediation model: definition



DAG Relton & Davey Smith (2012) A Step 1 SNP 1 Exposure CpG Phenotype B Step 2 SNP 2 Phenotype Exposure CpG $Y = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon_1,$ $X = \alpha_0 + \alpha_1 G \mathbf{1} + \varepsilon_2$ $Z = \gamma_0 + \gamma_1 G 2 + \gamma_2 X + \varepsilon_3$



$$\begin{pmatrix} arepsilon_1 \\ arepsilon_2 \\ arepsilon_3 \end{pmatrix} \sim \textit{MVN}(0, \Sigma)$$



Mediation model: Mag single study fit in OpenMx I

•
$$R_X^2 = 0.3, R_{Z,G2}^2 = 0.16$$



Mediation model: Math single study fit in OpenMx II

> mxTryHard(mediation.model)

- > summary(mediation.fit2)
- • •

free parameters:

	name	matrix	row	СС	l Estimate	Std.Error
1	beta1	A1	1	2	1.0109453	0.018136100
2	gamma1	A1	3	2	1.0209632	0.015113946
3	beta2	A1	1	3	1.0019530	0.009864446
4	alpha1	A1	2	4	1.0155065	0.015515166
5	gamma2	A1	3	5	1.0259136	0.013303523
6	errory	S1	1	1	0.9793607	0.027248906
7	errorxy	S1	1	2	0.7425581	0.020213100
8	errorx	S1	2	2	1.0150569	0.014356109
9	erroryz	S1	1	3	0.3855889	0.021258490
10	errorxz	S1	2	3	0.4930445	0.018988609
11	errorz	S1	3	3	0.9931549	0.020481237
12	varg1	S1	4	4	0.4218099	0.005965607
13	varg2	S1	5	5	0.4260051	0.006024941
14	beta0	M1	1	у	0.9849710	0.030509203
15	alpha0	M1	1	x	1.0037412	0.013839038
16	gamma0	M1	1	z	0.9588452	0.027641798
17	meang1	M1	1	g1	0.6115000	0.006494691
18	meang2	M1	1	g2	0.6088000	0.006526907

. . .

Mediation model: Lancaster & Lancaster Mathematics & Statistics University & Mathematics & Statistics & &



> fixed1 <- tssem1(scovs, sampsizes, method="FEM", cor.analysis=FALSE) > summary(fixed1) Coefficients: Estimate Std.Error z value Pr(>|z|) S[1,1] 14.0847613 0.2818307 49.9760 <2e-16 *** S[1,2] 4.1042813 0.0861058 47.6655 <2e-16 *** S[1,3] 7.0914238 0.1465197 48.3991 <2e-16 *** S[1,4] 0.8598434 0.0365179 23.5458 <2e-16 *** S[1,5] 0.6208533 0.0429352 14.4602 <2e-16 *** S[2,2] 1,4335084 0,0286838 49,9762 <2e-16 *** S[2,3] 1.9145515 0.0435214 43.9911 <2e-16 *** S[2,4] 0.4252767 0.0125247 33.9550 <2e-16 *** S[2,5] -0.0094234 0.0134083 -0.7028 0.4822 S[3,3] 4.0431484 0.0809034 49.9750 <2e-16 *** S[3,4] 0.4272514 0.0194131 22.0084 <2e-16 *** S[3,5] 0.6332306 0.0242343 26.1295 <2e-16 *** S[4,4] 0.4204346 0.0084129 49.9749 <2e-16 *** S[4,5] -0.0087257 0.0072621 -1.2015 0.2295 S[5,5] 0.6263648 0.0125336 49.9748 <2e-16 *** Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1



Mediation model: Mather & Sta meta-analysis – stage 2 covs

```
> fixed2 <- tssem2(fixed1, Amatrix=A1, Smatrix=S1, Fmatrix=F1,</p>
                 diag.constraints=TRUE, intervals.type="LB")
+
> summary(fixed2)
95% confidence intervals: Likelihood-based statistic
Coefficients:
      Estimate Std.Error lbound ubound z value Pr(>|z|)
beta1 1.02087
                     NA 0.99777 1.04377
                                           NΑ
                                                    NΑ
beta2 0.99634
                     NA 0.98214 1.01051
                                           NA
                                                    NA
                     NA 0.99349 1.02951
                                           NA
alpha1 1.01150
                                                    NA
gamma1 1.02570
                     NA 1.00810 1.04302
                                           NΑ
                                                    NΑ
gamma2 1.02683
                     NA 1.00763 1.04595
                                           NΑ
                                                    NΑ
       0.97624
                     NA 0.92966 1.02415
                                           NA
                                                    NA
ev
       0.73331
                     NA 0.69452 0.77253
                                           NΑ
                                                    NΑ
exv
                     NA 0.96390 1.04269
                                           NΑ
                                                    NΑ
ev
      1.00333
      0.35686
                     NA 0.32107 0.39343
                                           NA
                                                    NA
eyz
                     NA 0.41927 0.48883
exz
       0.45387
                                           NA
                                                    NA
                     NA 0.92302 1.00517
       0.96391
                                           NΑ
                                                    NΑ
67
```





- MASEM (inc. TSSEM) implemented in metaSEM R package
- Stage 1: estimate pooled correlation/covariance matrix
- Stage 2: SEM (ADF weighted by inverse Stage 1 var-cov)
- TSSEM can be used for MR analyses and complex mediation models
- sem (Stata) can't fit Stage 2
- Using correlations returns estimates for standardized variables
- Bayesian approach of Prevost *et al.* (2007) (Fisher's *z*)
- MR using path analysis could help uptake by researchers in psychology

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